2009. Spring: Electro-Optics  
(Prof. Sin-Doo Lee, Rm. 301-1109, [http://mipd.snu.ac.kr](http://mipd.snu.ac.kr))

*Optical Waves in Crystals*  

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| Mar. 02, 04 | * Basics of Crystal Structures  
Chap. 1: EM Fields | Apr. 30 | Periodic Medium II       |
| Mar. 09, 11 | Chap. 3: Polarization of Light  
(Tutorial) | May 04, 06 | Chap. 7: Electro-Optics I |
| Mar. 16, 18 | Chap. 4: EM Propagation I | May 11, 13 | Take-Home Exam           |
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| Apr. 13, 15 | Midterm Exam I                                     | Jun. 01, 03 | Chap. 11: Guided Optics |
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Chap. 0 Bonding and Types of Solids

0.1 Molecules and General Bonding Principles

- Net force = attractive and repulsive

Fig. 1.3: (a) Force vs interatomic separation and (b) Potential energy vs interatomic separation.

0.2 Covalently Bonded Solids: H₂, CH₄, diamond

Fig. 1.5: (a) Covalent bonding in methane, CH₄, involves four hydrogen atoms sharing electrons with one carbon atom. Each covalent bond has two shared electrons. The four bonds are identical and repel each other. (b) Schematic sketch of CH₄ on paper. (c) In three dimensions, due to symmetry, the bonds are directed towards the corners of a tetrahedron.

0.3 Metallic Bonding: electron gas or cloud (collective sharing of electrons)

Fig. 1.7: In metallic bonding the valence electrons from the metal atoms form a "cloud of electrons" which fills the space between the metal ions and "glues" the ions together through the coulombic attraction between the electron gas and positive metal ions.

0.4 Ionically Bonded Solids: salt (cation-anion)

Fig. 1.8: The formation of an ionic bond between Na and Cl atoms in NaCl. The attraction is due to coulombic forces.

- Potential energy per ion pair in solid NaCl

Fig. 1.10: Sketch of the potential energy per ion-pair in solid NaCl. Zero energy corresponds to neutral Na and Cl atoms infinitely separated.

0.5 Secondary Bonding: hydrogen bonds (polar), van der Waals bonds (induced dipolar)

Fig. 1.12: The origin of van der Waals bonding between water molecules. (a) The H₂O molecule is polar and has a net permanent dipole moment. (b) Attractions between the various dipole moments in water gives rise to van der Waals bonding.

Fig. 1.13: Induced dipole-induced dipole interaction and the resulting van der Waals force.

0.6 The Crystalline State

0.6.1 Type of Crystals: periodic array of points in space - lattice

Fig. 1.70: (a) A simple square lattice. The unit cell is a square with a side $a$. (b) Basis has two atoms. (c) Crystal = Lattice + Basis. The unit cell is a simple square with two atoms. (d) Placement of basis atoms in the crystal unit cell.
Fig. 1.71: The seven crystal systems (unit cell geometries) and fourteen Bravais lattices.
- Face-Centered Cubic (FCC) Structure:

Fig. 1.30: (a) The crystal structure of copper is Face Centered Cubic (FCC). The atoms are positioned at well defined sites arranged periodically and there is a long range order in the crystal. (b) An FCC unit cell with closed packed spheres. (c) Reduced sphere representation of the FCC unit cell. Examples: Ag, Al, Au, Ca, Cu, γ-Fe (>912°C), Ni, Pd, Pt, Rh
- Body-Centered Cubic (BCC) Structure:

Examples: Alkali metals (Li, Na, K, Rb), Cr, Mo, W, Mn, α-Fe (< 912°C), β-Ti (> 882°C).

Fig. 1.31: Body centered cubic (BCC) crystal structure. (a) A BCC unit cell with closely packed hard spheres representing the Fe atoms. (b) A reduced-sphere unit cell.
- Hexagonal Closed Packed (HCP) Structure:

Examples: Be, Mg, $\alpha$-Ti ($< 882^\circ$C), Cr, Co, Zn, Zr, Cd

Fig. 1.32: The Hexagonal Close Packed (HCP) Crystal Structure. (a) The Hexagonal Close Packed (HCP) Structure. A collection of many Zn atoms. Color difference distinguishes layers (stacks). (b) The stacking sequence of closely packed layers is ABAB (c) A unit cell with reduced spheres (d) The smallest unit cell with reduced spheres.
- Diamond & Zinc Blende Cubic Structure:

![Diamond Unit Cell](image1)

**Fig. 1.33**: The diamond unit cell is cubic. The cell has eight atoms. Grey Sn (α-Sn) and the elemental semiconductors Ge and Si have this crystal structure.

![Zinc Blende Crystal Structure](image2)

**Fig. 1.34**: The Zinc blende (ZnS) cubic crystal structure. Many important compound crystals have the zinc blende structure. Examples: AlAs, GaAs, GaP, GaSb, InAs, InP, InSb, ZnS, ZnTe.

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Table 1.3 Properties of some important crystal structures

<table>
<thead>
<tr>
<th>Crystal Structure</th>
<th>$a$ and $R$ ($R$ is the radius of the atom)</th>
<th>Coordination Number (CN)</th>
<th>Number of atoms per unit cell</th>
<th>Atomic Packing Factor</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple cubic</td>
<td>$a = 2R$</td>
<td>6</td>
<td>1</td>
<td>0.52</td>
<td>None</td>
</tr>
<tr>
<td>BCC</td>
<td>$a = 4R/\sqrt{3}$</td>
<td>8</td>
<td>2</td>
<td>0.68</td>
<td>Many metals: $\alpha$-Fe, Cr, Mo, W</td>
</tr>
<tr>
<td>FCC</td>
<td>$a = 4R/\sqrt{2}$</td>
<td>12</td>
<td>4</td>
<td>0.74</td>
<td>Many metals Ag, Au, Cu, Pt</td>
</tr>
</tbody>
</table>
| HCP               | $a = 2R$  
$c = 1.633a$                      | 12                       | 2                             | 0.74                 | Many metals: Co, Mg, Ti, Zn |
| Diamond           | $a = 8R/\sqrt{3}$                           | 4                        | 8                             | 0.34                 | Covalent solids: Diamond, Ge, Si, $\alpha$-Sn. |
| Zinc blende       | 4                                            | 8                        | 0.34                          | Many covalent and ionic solids. Many compound semiconductors. ZnS, GaAs, GaSb, InAs, InSb |
| NaCl              | 6                                            | 4 cations                | 0.67                          | Ionic solids such as NaCl, AgCl, LiF MgO, CaO |
|                   |                                              | 4 anions                 | (NaCl)                        | Ionic packing factor depends on relative sizes of ions. |
| CsCl              | 8                                            | 1 cation  
1 anion             |                               | Ionic solids such as CsCl, CsBr, CsI |

0.6.2 Three Phases of Carbon

(a) Diamond unit cell

(b) Graphite

(c) Buckminsterfullerene

Fig. 1.42: The three allotropes of carbon.
Chap. 1. Electromagnetic Fields

1.1. Maxwell's Equations and Wave Equations

- **E** and **H**: the electric and magnetic field vectors \((\mathbf{E}, \mathbf{H}) \leftrightarrow (\mathbf{D}, \mathbf{H})\)

  In MKS units, \(\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0\) with constitutive equations
  \[ \mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P} \]
  \[ \mathbf{B} = \mu \mathbf{H} = \mu_0 \mathbf{H} + \mathbf{M} \]
  \[ \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \]
  \[ \nabla \cdot \mathbf{D} = \rho \]
  \[ \nabla \cdot \mathbf{B} = 0 \]

- Poynting's Theorem and Conservation Law:
  \[ \frac{\partial U}{\partial t} + \nabla \cdot \mathbf{S} = - \mathbf{J} \cdot \mathbf{E} \]
  \[ U = \frac{1}{2} (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}) \quad \text{and} \quad \mathbf{S} = \mathbf{E} \times \mathbf{H} \]

  \(U\) = the energy density of the EM fields (joules/m\(^3\))
  \(\mathbf{S}\) = the Poynting vector, energy flow (joules/m\(^2\).sec)
  \(\nabla \cdot \mathbf{S}\) = the net EM power flowing out of a unit volume.

[Homework] Conservation of a linear momentum of the EM fields: Problem 1.4 (p. 18)
1.4. Wave Equations

- From Maxwell's equations, $\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{E} \right) + \frac{\partial}{\partial t} \nabla \times \mathbf{H} = 0$

  Differentiating the above eqn. wrt to time and eliminating $\mathbf{H}$,

  \[ \nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{E} \right) + \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \]

  Use the vector identities

  \[ \nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{E} \right) = \frac{1}{\mu} \nabla \times (\nabla \times \mathbf{E}) + \left( \nabla \frac{1}{\mu} \right) \times (\nabla \times \mathbf{E}) \]

  \[ \nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \]

  \[ \rightarrow \nabla^2 \mathbf{E} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \left( \nabla \log \mu \right) \times (\nabla \times \mathbf{E}) - \nabla (\nabla \cdot \mathbf{E}) = 0 \]

  Since $\nabla \cdot (\epsilon \mathbf{E}) = \epsilon \nabla \cdot \mathbf{E} + \mathbf{E} \cdot \nabla \epsilon$,

  \[ \nabla^2 \mathbf{E} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \left( \nabla \log \mu \right) \times (\nabla \times \mathbf{E}) + \nabla (\mathbf{E} \cdot \nabla \log \epsilon) = 0 \]

  Similarly, $\nabla^2 \mathbf{H} - \mu \epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} + \left( \nabla \log \epsilon \right) \times (\nabla \times \mathbf{H}) + \nabla (\mathbf{H} \cdot \nabla \log \mu) = 0$
- Inside a **homogeneous** and **isotropic** medium, $\nabla \log(\mu \text{ or } \epsilon) \to 0$

$$\nabla^2 \mathbf{E} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 ; \nabla^2 \mathbf{H} - \mu \epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \quad \rightarrow \quad \psi = e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \quad \text{with } |\mathbf{k}| = \omega \sqrt{\mu \epsilon}$$

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu \epsilon}} \rightarrow c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.997930 \times 10^8 \text{ m/s}$$

the refractive index of the medium $n = c/v = \sqrt{\mu \epsilon / \mu_0 \epsilon_0}$

1.5. Propagation of Laser Pulse; Group Velocity

- Denoting $A(k) =$ the amplitude of the plane-wave component with $k$,

$$\psi(z,t) = \int_{-\infty}^{\infty} A(k) e^{i[\omega(k)t - kz]} dk \quad \text{with } |A(k)|^2 = \text{the Fourier spectrum}$$

- For a laser pulse, $\omega(k) = \omega_0 + \left(\frac{d\omega}{dk}\right)_0 (k - k_0) + \ldots$ (see the next page)

then, $\psi(z,t) \approx e^{i(\omega_0 t - k_0 z)} \int_{-\infty}^{\infty} A(k) \exp \left\{ i \left[ \left(\frac{d\omega}{dk}\right)_0 t - z \right] (k - k_0) \right\} dk$
\[ = e^{i(\omega_0 t - k_0 z)} \phi \left[ z - \left( \frac{d\omega}{dk} \right)_0 t \right] \text{ with the envelope function } \phi \]

- The laser pulse travels along undistorted in shape with a velocity 

\[ v_g = \left( \frac{d\omega}{dk} \right)_0 \text{; the group velocity} \]
- In optics, $k = n(\omega)\frac{\omega}{c}$, the phase velocity $v_p = \frac{c}{n(\omega)}$, $v_g = \frac{c}{n + \omega(\frac{dn}{d\omega})}$
Chap. 3. Polarization of Light Waves

3.2. Polarization of Monochromatic Plane Waves

- In a complex-function representation,

\[ E(z, t) = \text{Re}[Ae^{i(\omega t - kz)}] \rightarrow E_x = A_x \cos(\omega t - kz + \delta_x) \text{ where } A = \hat{x}A_x e^{i\delta_x} + \hat{y}A_y e^{i\delta_y} \]

\[ E_y = A_y \cos(\omega t - kz + \delta_y) \]

In the \( x-y \) plane,

\[ \left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 - \frac{\cos\delta}{A_x A_y} E_x E_y = \sin^2\delta \text{ where } \delta = \delta_y - \delta_x \quad (-\pi < \delta < \pi) \]

- Let \( x' \) and \( y' \) be a new set of axes along the principal axes of the ellipse:

\[ \left(\frac{E_{x'}}{a}\right)^2 + \left(\frac{E_{y'}}{b}\right)^2 = 1, \text{ with the rotation angle } \phi \]

where \( a^2 = A_x^2 \cos^2\phi + A_y^2 \sin^2\phi + 2A_x A_y \cos\delta \cos\phi \sin\phi \) and \( \tan2\phi = \frac{2A_x A_y}{A_x^2 - A_y^2} \cos\delta \)

\[ b^2 = A_x^2 \sin^2\phi + A_y^2 \cos^2\phi - 2A_x A_y \cos\delta \cos\phi \sin\phi \]

\[ \frac{2A_x A_y}{A_x^2 - A_y^2} \cos\delta \]
3.2.1 Linear and Circular Polarization

- *Linearly* polarized: straight lines, \( \delta = \delta_y - \delta_x = m\pi \) \((m = 0, 1)\) and \( \frac{E_y}{E_x} = (-1)^m \frac{A_y}{A_x} \)
- *Circularly* polarized: right-handed \( \delta = -\pi/2 \) and left-handed \( \delta = +\pi/2; \) \( A_y = A_x \)
- Ellipticity of polarization: \( e = \pm \frac{b}{a} \), (+) when the rotation of \( E \) is right-handed.

\[
\chi = e^{i\delta} \tan \psi = \frac{A_y}{A_x} e^{i(\delta_y - \delta_x)}
\]
Figure 3.2. Polarization ellipses at various phase angles $\theta$, where: (a) $E_x = \cos(\omega t - kz)$, $E_y = \cos(\omega t - kx + \delta)$; (b) $E_x = \frac{1}{2} \cos(\omega t - kx)$, $E_y = \cos(\omega t - kx + \delta)$.

Figure 3.3. Each point of the complex plane is associated with a polarization state.
3.3 Complex-Number Presentation

- The amplitudes and the phase angles of the $x$ and $y$ components of $E$:

$$\chi = e^{i\delta} \tan \psi = \frac{A_y}{A_x} e^{i(\delta_y - \delta_x)}$$

- The inclination angle $\phi$ and the ellipticity angle $\theta (\equiv \tan^{-1} e)$ of the polarization ellipse:

$$\tan 2\phi = \frac{2 Re[\chi]}{1 - |\chi|^2} \quad \text{and} \quad \sin 2\theta = -\frac{2 Im[\chi]}{1 + |\chi|^2}$$

3.4 Jones Vector Presentation

- 1941, R. C. Jones,

$$\vec{E}(z,t) = Re[\vec{A} e^{i(\omega t - kz)}] \rightarrow \vec{J} = \begin{pmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{pmatrix} \text{ with normalization } \vec{J}^* \cdot \vec{J} = 1$$

- Mutually orthogonal polarization:

$$\begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix} \Leftrightarrow \begin{pmatrix} -\sin \psi \\ \cos \psi \end{pmatrix}, \quad \hat{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Leftrightarrow \hat{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \Leftrightarrow \hat{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}; \quad \hat{R}^* \cdot \hat{L} = 0$$
<table>
<thead>
<tr>
<th>Polarization Ellipse</th>
<th>Jones Vector</th>
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<tbody>
<tr>
<td></td>
<td>$\begin{pmatrix} 1 \ 0 \end{pmatrix}$</td>
</tr>
<tr>
<td></td>
<td>$\begin{pmatrix} 0 \ 1 \end{pmatrix}$</td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \ 1 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \ -1 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \ 1 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \ 2 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \ -2 \end{pmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>
\[ \hat{R} = \frac{1}{\sqrt{2}} (\hat{x} - i\hat{y}), \quad \hat{L} = \frac{1}{\sqrt{2}} (\hat{x} + i\hat{y}) \]
\[ \hat{x} = \frac{1}{\sqrt{2}} (\hat{R} + \hat{L}), \quad \hat{y} = \frac{i}{\sqrt{2}} (\hat{R} - \hat{L}) \]

- For a general elliptical polarization, \( \mathcal{J}(\psi, \phi) = \begin{pmatrix} \cos \psi \\ e^{i\delta} \sin \psi \end{pmatrix} \)