Chap. 6. Electromagnetic Propagation in Periodic Media

- The propagation of em radiation in periodic media: x-ray diffraction in crystals, light diffraction from the periodic strain variation accompanying a sound wave ("phonon"), the "forbidden band" of light in periodic layered media

- Diffraction gratings, holograms, free-electron lasers, distributed-feedback (DFB) lasers, distributed-Bragg-reflectors (DBR) lasers, high-reflectance Bragg mirrors, acousto-optic filters

- A formal analogy to the quantum theory of electrons in crystals, the concepts of the Bloch waves, forbidden gaps, evanescent waves and surface waves

6.1. Periodic Media

- The optically periodic medium: described in terms of the dielectric and permeability tensors.

  the translational symmetry of the medium: periodic function of \( x \)

  \[ \varepsilon(x) = \varepsilon(x + a), \quad \mu(x) = \mu(x + a) \quad \text{with} \quad a = \text{the lattice vector} \]

- The propagation of monochromatic (frequency \( \omega \)) laser radiation in a periodic medium:

  \[ \nabla \times \mathbf{H} = i\omega\varepsilon \mathbf{E}, \quad \nabla \times \mathbf{E} = -i\omega\mu \mathbf{H} \]

  invariant under \( x \rightarrow x + a \) in the operator \( \nabla \) and \( \varepsilon, \mu \)
- The normal modes with the translational symmetry: Floquet (or Bloch) theorem

\[ E = E_K(x) e^{-iK \cdot x}, \quad H = H_K(x) e^{-iK \cdot x} \]

where \( E_K(x), H_K(x) \) are both periodic in \( x \), \( E_K(x) = E_K(x + a) \), \( H_K(x) = H_K(x + a) \)

- The dispersion relation: \( \omega = \omega(K) \), \( K \) = the Bloch wave vector.

- Determine \( E_K(x), H_K(x) \), and \( \omega(K) \)

6.1.1. One-Dimensional Periodic Media

- 1-dimensional periodic optical medium: \( \epsilon(z) = \epsilon(z + l\Lambda) \) with \( \Lambda \) = the period, \( l \) = an integer

- Let \( \theta \) be the angle of incidence, then the constructive interference in reflection occurs when

\[ m\lambda = 2\Lambda \cos\theta \quad (\text{Bragg condition}) \]

- The propagation of em radiation in this medium obeys the wave equation

\[ \nabla \times (\nabla \times E) - \omega^2 \mu \epsilon E = 0 \]
- For a periodic medium, \( \varepsilon(x) = \sum_G \varepsilon \, e^{-i \mathbf{G} \cdot \mathbf{x}} \) where \( \mathbf{G} \) runs over all the reciprocal lattice vectors, including \( \mathbf{G} = \mathbf{0} \): in 1-dimension, \( \mathbf{G} = lg = l \frac{2\pi}{\Lambda} \hat{z}, \quad l = 0, \pm 1, \pm 2, \pm 3, \ldots \)

Then, \( \varepsilon(z) = \sum_l \varepsilon_l e^{-il(2\pi/\Lambda)z} \)

- The electric field in the periodic medium as a Fourier integral: \( \mathbf{E} = \int d^3k \mathbf{A}(k) e^{-ik \cdot \mathbf{x}} \)

- The resulting wave equation in the periodic medium:

\[
\int d^3k \times [k \times \mathbf{A}(k)] e^{-ik \cdot \mathbf{x}} = \omega^2 \mu \sum_G \int d^3k \varepsilon \mathbf{G} \mathbf{A}(k - \mathbf{G}) e^{-ik \cdot \mathbf{x}}
\]

\[\rightarrow k \times [k \times \mathbf{A}(k)] + \omega^2 \mu \sum_G \varepsilon \mathbf{G} \mathbf{A}(k - \mathbf{G}) = 0 \text{ for all } \mathbf{k} \]

This is an infinite set of homogeneous equations for the unknown coefficients \( \mathbf{A}(k) \).

*Not all the coefficients \( \mathbf{A}(k) \) are coupled, only the coefficients of the form \( \mathbf{A}(k-G) \) are coupled \( \rightarrow \) the whole set into many subsets, each labeled by a wave vector \( \mathbf{K} \) and containing equations that involve \( \mathbf{A}(\mathbf{K}) \) and \( \mathbf{A}(\mathbf{K}-\mathbf{G}) \) with all possible \( \mathbf{G} \)'s.
- The solution of a subset labeled by $K$ can be written as

$$E_K = \sum_G A(K - G) e^{-i(K-G) \cdot r}$$

$$= e^{-iK \cdot r} \sum_G A(K - G) e^{iG \cdot r} = e^{-iK \cdot r} E_K(r)$$

In 1-dim. case, $G = l \hat{g} = l2\pi \hat{z}/\Lambda$, so that $E_K(r) = \sum \lambda A(K - l \hat{g}) e^{i(2\pi/\Lambda) z}$, periodic in $\Lambda$.

- For given frequency $\omega$, find the K vector from $k \times [k \times A(k)] + \omega^2 \mu \sum G \epsilon G A(k - G) = 0$

Assuming that the medium is homogeneous along the x- and y- directions,

the Bloch mode of the electric field $E = e^{-i(K_x x + K_y y)} e^{-iK_z z} E_K(z)$

There exist regions of $\omega$ where $K_z$ becomes a complex number, so that the Bloch wave becomes evanescent: "forbidden bands" of the periodic medium.

→ Incident radiation under these Bragg reflection conditions will be totally reflected.

- Devices based on the periodic medium NEAR or IN the forbidden bands.
- Approximate solution for the Bloch waves NEAR the Bragg condition:
Assuming that the wave propagates along the $z$-axis ($K_x = K_y = 0$), and the field vector is transverse to the propagation vector (i.e., $\mathbf{k} \cdot \mathbf{E} = 0$) and the medium is isotropic,

Then, $k^2 A(k) - \omega^2 \mu \sum_l \epsilon_l A(k - lg) = 0$

- To find the Bloch wave with wave number $K$, we need to solve the set of the above equations with $k = K, K \pm g, K \pm 2g, K \pm 3g, \ldots$.

Writing the first few terms in the above equation for $k = K$,

$$K^2 A(K) - \omega^2 \mu \epsilon_0 A(K) - \omega^2 \mu \epsilon_1 A(K - g) - \omega^2 \mu \epsilon_{-1} A(K + g) - \ldots = 0$$

$$\rightarrow A(K) = \frac{1}{K^2 - \omega^2 \mu \epsilon_0} \left[ \omega^2 \mu \epsilon_1 A(K - g) + \omega^2 \mu \epsilon_{-1} A(K + g) + \ldots \right]$$

Similarly, for $k = K - g$ and $k = K + g$,

$$A(K - g) = \frac{1}{(K - g)^2 - \omega^2 \mu \epsilon_0} \left[ \omega^2 \mu \epsilon_1 A(K - 2g) + \omega^2 \mu \epsilon_{-1} A(K) + \ldots \right] \text{ and}$$

$$A(K + g) = \frac{1}{(K + g)^2 - \omega^2 \mu \epsilon_0} \left[ \omega^2 \mu \epsilon_1 A(K) + \omega^2 \mu \epsilon_{-1} A(K + 2g) + \ldots \right]$$
- If $|K - g| \approx K$ (Bragg condition) and $K^2 \approx \omega^2 \mu \epsilon_0$ in $A(\mathbf{K}), A(\mathbf{K} - g)$, $A(\mathbf{K} + g)$, the dominant terms are $A(\mathbf{K})$ and $A(\mathbf{K} - g)$: the plane wave components $A(\mathbf{K})$ and $A(\mathbf{K} - g)$ are resonantly coupled.

Neglecting all other coefficients, the series equations $k^2 A(k) - \omega^2 \mu \sum_l \epsilon_l A(k - lg) = 0$ for the the Bloch wave with wave number $\mathbf{K}$ becomes

$$
(K^2 - \omega^2 \mu \epsilon_0) A(\mathbf{K}) - \omega^2 \mu \epsilon_1 A(\mathbf{K} - g) = 0
$$

$$
-\omega^2 \mu \epsilon_{-1} A(\mathbf{K}) + [(K - g)^2 - \omega^2 \mu \epsilon_0] A(\mathbf{K} - g) = 0
$$

$\rightarrow$ Nontrivial solution: \[ \det \begin{vmatrix} K^2 - \omega^2 \mu \epsilon_0 & -\omega^2 \mu \epsilon_1 \\ -\omega^2 \mu \epsilon_{-1} & (K - g)^2 - \omega^2 \mu \epsilon_0 \end{vmatrix} = 0 \]

or $(K^2 - \omega^2 \mu \epsilon_0)[(K - g)^2 - \omega^2 \mu \epsilon_0] - (\omega^2 \mu |\epsilon_1|)^2 = 0$ where \( \text{real} \ \epsilon_{-1} = \epsilon_1^* \)

The Bragg condition is satisfied exactly at $\mathbf{K} = g/2 = \pi/\Lambda$; at this $\mathbf{K}$ value,

the roots for $\omega^2$: $\omega^2 = \frac{K^2}{\mu (\epsilon_0 \pm |\epsilon_1|)}$; spectral band edges

Between $\omega_+ \sim \omega_-$, the roots for $\mathbf{K}$ are complex: evanescent, "forbidden band"
Figure 6.3. (a) Dispersion relation near a forbidden band. We notice that $K$ becomes complex when the real part of $\alpha A$ is $\nu$. The dispersion relation is plotted for the periodic structure shown in Fig. 6.3 with $\alpha_1 = 3.2$, $\alpha_2 = 3.4$, $a = b = 0.5\lambda$. (b) A general dispersion relation $\omega(K)$ showing the fundamental ($l = 1$) and higher-order ($l = 2, 3$) forbidden bands.
At frequencies \( \omega \) outside the forbidden band, the roots of
\[
(K^2 - \omega^2 \mu \varepsilon_0) \left[ (K - g)^2 - \omega^2 \mu \varepsilon_0 \right] - (\omega^2 \mu |\varepsilon_1|)^2 = 0
\]
are real; propagating waves.

- The relation between \( \omega \) and \( K \): the dispersion relation.

*Waves in the forbidden bands do not propagate due to Bragg reflection.

Let us see the wave number \( K \) at the center of the forbidden band

when \( \omega^2 = \left( \frac{1}{2} g \right)^2 / \mu \varepsilon_0 \) [see, \( |K - g| \approx K, \ K^2 \approx \omega^2 \mu \varepsilon_0 \)], and \( K = \frac{1}{2} g + x \) where \( |x| \ll \frac{1}{2} g \)

then,
\[
(K^2 - \omega^2 \mu \varepsilon_0) \left[ (K - g)^2 - \omega^2 \mu \varepsilon_0 \right] - (\omega^2 \mu |\varepsilon_1|)^2 = 0
\]
becomes

\[
\rightarrow \ g^2 x^2 + \left( \frac{|\varepsilon_1|}{\varepsilon_0} \right)^2 \left( \frac{1}{4} g^2 \right)^2 = 0 , \text{ neglecting the term in } x^4;
\]
solving the above equation, the wave vector number \( K \) at the center of the forbidden band: \( K = \frac{1}{2} g \left( 1 \pm i \frac{|\varepsilon_1|}{2 \varepsilon_0} \right) \); complex, an exponentially decaying amplitude!

*This spatial attenuation is governed by the Fourier expansion coefficient \( \varepsilon_1 \)
- The forbidden band gap: $\Delta \omega_{\text{gap}} = |\omega_+ - \omega_-| \approx \frac{|\epsilon_i|}{\epsilon_0}$, proportional to the magnitude of the Fourier expansion coefficient of the dielectric constant.

*In terms of $\Delta \omega_{\text{gap}}$, the imaginary part of the wave number at the center of the forbidden band is proportional to $\frac{\Delta \omega_{\text{gap}}}{\omega}$; $k_i = \frac{1}{4} g \left( \frac{\Delta \omega_{\text{gap}}}{\omega} \right)$ where $g = \frac{2\pi}{\Lambda}$

- In general, the forbidden bands associated with the Bragg reflections are governed by all the Fourier expansion coefficients, $\epsilon_i$'s, of the dielectric function $\epsilon(z)$:

$$|K - lg| \approx K \ (l = \pm 1, \pm 2, \ldots) \quad \text{and} \quad K^2 \approx \omega^2 \mu \epsilon_0$$

then, the Bloch wave with wave number $K$ becomes

$$\left( K^2 - \omega^2 \mu \epsilon_0 \right) A(K) - \omega^2 \mu \epsilon_i A(K - lg) = 0$$

$$- \omega^2 \mu \epsilon_{-l} A(K) + [(K - lg)^2 - \omega^2 \mu \epsilon_0] A(K - lg) = 0$$

higher-order forbidden bands at $K = l \frac{g}{2} + l \frac{\pi}{\Lambda}$ with $\epsilon_{-l} = \epsilon_i^*$, so that $\Delta \omega_{\text{gap}} = \frac{\omega |\epsilon_i|}{\epsilon_0}$

- In many cases, $\epsilon_i$ becomes small as $l$ increases ($\epsilon_i \to 0$ as $l \to \infty$)

- For a general direction of propagation ($K_x, K_y \neq 0$), the dispersion is more complicated and depends on the polarization state.
6.2. Periodic Layer Structure

- The simple cases: alternating layers of transparent materials with different refractive indices.
- For a simplest two different materials with a refractive index profile given as

\[
n(z) = \begin{cases} 
  n_2, & 0 < z < b \\
  n_1, & b < z < \Lambda 
\end{cases} \quad \text{with} \quad n(z) = n(z + \Lambda)
\]

Figure 6.3. A schematic drawing of a periodic layered medium and the plane-wave amplitudes associated with the \( n \)th unit cell and its neighboring layers.
- In solving for the electric field vectors of the Bloch wave, \( \mathbf{E}(z) e^{i(\omega t - k_y y)} \) where we assumed that the plane of propagation is the \( yz \)-plane; \( k_y \) is the \( y \) component of the wave propagation vector, which remains constant throughout the medium.

The electric field within each homogeneous layer can be expressed as a sum of an incident and a reflected plane wave; the complex amplitudes of these two waves constitute the components of a column vector. In layer \( \alpha (\alpha = 1, 2) \) of the \( n \)-th unit cell,

the electric field represented by a column vector \( \begin{pmatrix} a_n^{(\alpha)} \\ b_n^{(\alpha)} \end{pmatrix}, \ \alpha = 1, 2 \)

- The electric-field distribution in the same layer:

\[
E(y, z) = [a_n^{(\alpha)} e^{-ik_{az}(z-nA)} + b_n^{(\alpha)} e^{ik_{az}(z-nA)}] e^{-ik_y y} \text{ with } k_{az} = \left[ \left( \frac{n_\alpha \omega}{c} \right)^2 - k_y^2 \right]^{1/2}, \ \alpha = 1, 2.
\]

The column vectors are NOT independent of each other. They are related through the continuity conditions at interfaces \( \rightarrow \) Only one vector (or two components of different vectors) can be chosen arbitrarily.

- In the case of TE waves (\( \mathbf{E} \) perpendicular to \( yz \) plane), imposing the continuity of "\( E_x \)" and \( H_y (H_y \propto \partial E_x / \partial z) \) at the interfaces, \( z = (n-1)A, \quad \frac{\partial E_x}{\partial z} = (n-1)A + b \), (see Fig. 6.3) leads to 4 equations:
\[ a_{n-1} + b_{n-1} = e^{ik_{2z}A} c_n + e^{-ik_{2z}A} d_n \]
\[ ik_{1z}(a_{n-1} - b_{n-1}) = ik_{2z}(e^{ik_{2z}A} c_n - e^{-ik_{2z}A} d_n) \]
\[ e^{ik_{2z}a} c_n + e^{-ik_{2z}a} d_n = e^{ik_{1z}a} a_n + e^{-k_{1z}a} b_n \]
\[ ik_{2z}(e^{ik_{2z}a} c_n - e^{-k_{2z}a} d_n) = ik_{1z}(e^{ik_{1z}a} a_n - e^{-ik_{1z}a} b_n) \]

In the matrix form,
\[
\begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}
\begin{pmatrix}
a_{n-1} \\
b_{n-1}
\end{pmatrix}
= \begin{pmatrix}
e^{ik_{2z}A} & e^{-ik_{2z}A} \\
e^{-k_{2z}A} & e^{k_{2z}A}
\end{pmatrix}
\begin{pmatrix}
c_n \\
c_n
\end{pmatrix},
\]

\[
\begin{pmatrix}
e^{ik_{2z}a} & e^{-ik_{2z}a} \\
e^{-k_{2z}a} & e^{k_{2z}a}
\end{pmatrix}
\begin{pmatrix}
c_n \\
d_n
\end{pmatrix}
= \begin{pmatrix}
e^{ik_{1z}a} & e^{-ik_{1z}a} \\
e^{-k_{1z}a} & e^{k_{1z}a}
\end{pmatrix}
\begin{pmatrix}
a_n \\
b_n
\end{pmatrix},
\]

where \( a_n \equiv a_n^{(1)} \), \( b_n \equiv b_n^{(1)} \), \( c_n \equiv a_n^{(2)} \), \( d_n \equiv b_n^{(2)} \)

Eliminating \( \begin{pmatrix} c_n \\ d_n \end{pmatrix} \), the matrix equation \( \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \); a unit-cell translation matrix. (recursion relation)

unimodular: \( AD - BC = 1 \)
The matrix elements are

\[
A = e^{ik_{1z}a} \left[ \cos k_{2z} b + \frac{1}{2} i \left( \frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}} \right) \sin k_{2z} b \right]
\]

\[
B = e^{-ik_{1z}a} \left[ \frac{1}{2} i \left( \frac{k_{2z}}{k_{1z}} - \frac{k_{1z}}{k_{2z}} \right) \sin k_{2z} b \right]
\]

\[
C = e^{ik_{1z}a} \left[ -\frac{1}{2} i \left( \frac{k_{2z}}{k_{1z}} - \frac{k_{1z}}{k_{2z}} \right) \sin k_{2z} b \right]
\]

\[
D = e^{-ik_{1z}a} \left[ \cos k_{2z} b - \frac{1}{2} i \left( \frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}} \right) \sin k_{2z} b \right]
\]

- For TM waves \((H\perp \text{yz plane})\),

\[
A_{TM} = e^{ik_{1z}a} \left[ \cos k_{2z} b + \frac{1}{2} i \left( \frac{n_{2}^{2}k_{1z}}{n_{1}^{2}k_{2z}} + \frac{n_{1}^{2}k_{2z}}{n_{2}^{2}k_{1z}} \right) \sin k_{2z} b \right]
\]

\[
B_{TM} = e^{-ik_{1z}a} \left[ \frac{1}{2} i \left( \frac{n_{2}^{2}k_{1z}}{n_{1}^{2}k_{2z}} - \frac{n_{1}^{2}k_{2z}}{n_{2}^{2}k_{1z}} \right) \sin k_{2z} b \right]
\]

\[
C_{TM} = e^{ik_{1z}a} \left[ -\frac{1}{2} i \left( \frac{n_{2}^{2}k_{1z}}{n_{1}^{2}k_{2z}} - \frac{n_{1}^{2}k_{2z}}{n_{2}^{2}k_{1z}} \right) \sin k_{2z} b \right]
\]

\[
D_{TM} = e^{-ik_{1z}a} \left[ \cos k_{2z} b - \frac{1}{2} i \left( \frac{n_{2}^{2}k_{1z}}{n_{1}^{2}k_{2z}} + \frac{n_{1}^{2}k_{2z}}{n_{2}^{2}k_{1z}} \right) \sin k_{2z} b \right]
\]
- The column vector of layer 1 in the zero-th unit cell, the remaining column vectors of the equivalent layers are related to that of the zero-th unit cell by
\[
\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = (AB)^n \begin{pmatrix} a_n \\ b_n \end{pmatrix} \rightarrow \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \left( (AB)^{-1} \right)^n \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-n} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} \text{ with } (AB)^{-1} = \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}
\]

or
\[
\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}^n \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}
\]

6.2.1. Block Waves and Band Structure

- A periodic layered medium: equivalent to a one-dimensional crystal which is invariant under lattice translations.

Bloch theorem: 
\[
E = E_K(z) e^{-iKz} e^{i(\omega t - k_y)}
\]

with \( E_K(z) = E_K(z + \Lambda) \); \( K \)=the Bloch wave number

[Question] How to determine \( K \) and \( E_K(z) \) as functions of \( \omega \) and \( k_y \) (Note that \( k_y = K_y \))

The periodic condition for the Bloch theorem:
\[
\begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{-i\Lambda} \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix}
\]
The column vector of the Bloch wave satisfies the eigenvalue equation:

\[
\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{iKA} \begin{pmatrix} a_n \\ b_n \end{pmatrix}; \quad \text{eigenvalue } e^{iKA}
\]

\[
\begin{vmatrix} A - e^{iKA} & B \\ C & D - e^{iKA} \end{vmatrix} = 0 \quad \rightarrow \quad e^{iKA} = \frac{1}{2} (A + D) \pm \left\{ \frac{1}{2} (A + D)^2 - 1 \right\}^{1/2}
\]

The eigenvectors \( \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} B \\ e^{iKA} - A \end{pmatrix} \) and

the corresponding column vectors for the \( n \)-th unit cell are \( \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{-iKA} \begin{pmatrix} B \\ e^{iKA} - A \end{pmatrix} \)

*The two eigenvalues of \( e^{iKA} \) are reciprocals of each other since the translation matrix is unimodular.

The dispersion relation between \( \omega, k_y \), and \( K \) for the Bloch wave function:

\[
K(k_y, \omega) = \frac{1}{A} \cos^{-1} \left[ \frac{1}{2} (A + D) \right]
\]

i) For \( |(1/2)(A \pm D)| < 1 \); real \( K \), thus propagating Bloch waves

ii) For \( |(1/2)(A + D)| > 1 \); \( K = m\pi/A + iK_i \) (complex), evanescent, "forbidden" band

iii) \( |(1/2)(A + D)| = 1 \) at band-edge frequencies
- The final result for the Bloch wave in layer 1 of the $n$-th unit cell:

$$E_K(z)e^{-iKz} = \left[ a_0 e^{-ik_{1z}(z-nA)} + b_0 e^{ik_{1z}(z-nA)} \right] e^{iK(z-nA)} e^{-iKz}$$

6.3. Bragg Reflection

- Consider a periodic medium layered with $N$ unit cells
- The coefficient of reflection: \( r_N = \left( \frac{b_0}{a_0} \right) \) and use \( \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \left( \begin{array}{cc} A & B \\ C & D \end{array} \right)^N \begin{pmatrix} a_N \\ b_N \end{pmatrix} \)

\[
\left( \begin{array}{cc} A & B \\ C & D \end{array} \right)^N = \begin{pmatrix} AU_{N-1} - U_{N-2} & BU_{N-1} \\ CU_{N-1} & DU_{N-1} - U_{N-2} \end{pmatrix}
\]

where \( U_N = \frac{\sin(N+1)KA}{\sin KA} \)

with \( K(k_y, \omega) = \frac{1}{A} \cos^{-1} \left[ \frac{1}{2} (A + D) \right] \)

Then, \( r_N = \frac{CU_{N-1}}{AU_{N-1} - U_{N-2}} \) (see Prob. 6.20), the reflectivity \( |r_N|^2 = \frac{|C|^2}{|C|^2 + \left( \frac{\sin KA}{\sin NKA} \right)^2} \)

[Homework] Prob. 6.10: Transmission coefficient of the Bragg reflector

- Considering the reflectivity of a single unit cell, \( |r_1|^2 = \frac{|C|^2}{|C|^2 + 1} \) or \( |C|^2 = \frac{|r_1|^2}{1 - |r_1|^2} \).
  
  For a typical Bragg reflector, \( |r_1|^2 \ll 1 \rightarrow |C|^2 \sim |r_1|^2 \)

  At the band edges, \( KA = m\pi \), the reflectivity \( |r_M|^2 = \frac{|C|^2}{|C|^2 + (1/N)^2} \)
6.4. Coupled Mode Theory