OPTIMIZATION OF A REFLECTIVE ANTIFERROELECTRIC LIQUID CRYSTAL DISPLAY USING THE 2 × 2 JONES MATRIX

Chang-Jae Yu, Doo-Hwan You, Jeong-Ho Son, and Sin-Doo Lee
School of Electrical Engineering #032, Seoul National University,
Seoul 151–742, Korea

We report on the design optimization of a reflective antiferroelectric liquid crystal (AFLC) display in the in-plane optical geometry within the framework of the 2 × 2 Jones matrix formalism. For obtaining good achromaticity and high brightness in the reflective configuration, cell parameters of the AFLC layer are optimized in the geometry such that the average optic axis of the AFLC layer lies on the plane parallel to both substrates.

Keywords: 2 × 2 Jones matrix; achromaticity; antiferroelectric liquid crystal; reflective display; viewing angle

1. INTRODUCTION

A reflective liquid crystal display (LCD) with no backlight unit has attracted great interest for portable, low power consumption applications. Although various reflective modes [1–3] based on nematic LCs have been studied and optimized to improve brightness, contrast, and achromaticity [4,5], these nematic-based reflective LCDs still suffer from slow response for practical applications.

Recently, a reflective LCD [6] with a single polarizer and a retardation film using an antiferroelectric liquid crystal (AFLC) has been proposed to obtain the fast response for video-rate applications. Moreover, quasi-achromatic and wide viewing properties of a reflective LCD [7] were reported in the in-plane optical geometry such that the average optic axis of the AFLC layer lies on the plane parallel to both substrates.

This work was supported in part by Ministry of Information and Communication through Advanced Backbone IT technology development project.

Address correspondence to Chang-Jae Yu, School of Electrical Engineering #032, Seoul National University, Seoul 151–742, Korea.
In this work, cell parameters of a reflective AFLC cell in the geometry such that the average optic axis of the cell changes on the plane parallel to both substrates are optimized to obtain high brightness and good achromaticity. Under the optimal condition, the viewing properties of the reflective AFLC cell are numerically calculated using the extended $2 \times 2$ Jones matrix method [8]. Reflectance of the AFLC cell are experimentally measured as a function of the applied electric field and theoretically predicted in a bilayer model [9].

2. OPTIMAL DESIGN OF THE AFLC CELL

The reflective AFLC cell being studied is composed of a single polarizer, a AFLC layer, a wide-band quarter (WQ) wave plate, and a reflector as shown in Figure 1. The WQ wave plate practically consists of a half wave (HW) and a quarter wave (QW) plates for fixed wavelength of 550 nm. In the entire range of visible light, in order to achieve a completely dark state, the optic axes of HW and QW plates make angles of $75^\circ$ and $15^\circ$ to that of the polarizer, respectively.

The electro-optical (EO) modulation is produced by the rotation of the average optic axis of the AFLC layer. For light propagating normally through a stack of uniform layers as shown in Figure 1, the $2 \times 2$ Jones matrix formalism is accurate enough to calculate the optical transmission as well as the reflection. For normal incidence, the reflectance of the cell is the square of a non-zero component of the Jones matrix through the

![FIGURE 1 In-plane optical geometry of a reflective AFLC cell.](image)
AFLC cell [7] and depends on the rotation of the average optic axis of the AFLC layer, $\theta_{LC}$, and the effective phase retardation through the AFLC layer, $\gamma_{LC} = 2\pi \Delta n_{LC} d_{LC} / \lambda$ where $\Delta n_{LC}$, $d_{LC}$, and $\lambda$ are the optical birefringence of the AFLC layer, the thickness of the layer, and the wavelength of incident light in vacuum, respectively.

Figure 2 shows a two-dimensional (2-D) gray scale map of the iso-reflectance in the reflective AFLC cell given in the plane spanned by $\theta_{LC}$ and $\gamma_{LC}$ for normal incidence. The reflectance along the $\theta_{LC}$-axis is symmetric to $\theta_{LC} = 45^\circ$ and that along the $\gamma_{LC}$-axis is symmetric to $\gamma_{LC} = 180^\circ$. The maximum reflectance occurs in the range of $90^\circ \leq \gamma_{LC} \leq 270^\circ$ and $22.5^\circ \leq \theta_{LC} \leq 67.5^\circ$. Between $22.5^\circ$ and $27^\circ$, high reflectance is obtained in a wide range of retardation $\gamma_{LC}$, implying that the thickness-tolerance and the achromaticity of the cell are improved.

In a simple case of normal incidence, we can assume that the reflectance depends only on the retardation of the AFLC layer in the entire range of visible light. For the ideal QW plate rotated from the polarizer by $45^\circ$, the reflectance is written as

$$R = 1 - \left[ \cos^2 2\theta_{LC} + \sin^2 2\theta_{LC} \cos \gamma_{LC} \right]^2. \quad (1)$$

The above analytical expression gives the reflectance map shown in Figure 2. Moreover, the optimal conditions for the rotation angle, $\theta_{LC}^{MAX}$,
and the retardation, $\gamma_{LC}^{\text{MAX}}$, through the AFLC layer can be obtained using $R=1$ in Eq. (1).

$$\gamma_{LC}^{\text{MAX}} = \cos^{-1}\left(-\cot^2 2\theta_{LC}^{\text{MAX}}\right) + 2m\pi,$$

where $m\pi/2 + \pi/8 \leq \theta_{LC}^{\text{MAX}} \leq m\pi/2 + 3\pi/8$. Here, $m$ and $n$ are integers ($0, \pm 1, \pm 2, \ldots$). The condition that $n=0$ and $m=0$ in the bounds of Eq. (2) gives the black solid line in Figure 2.

3. OPTICAL PROPERTIES UNDER THE OPTIMAL CONDITIONS

In a reflective AFLC cell, consisting of the AFLC layer and the WQ wave plate, we calculate the optical dispersion of the reflectance. Once the maximum reflectance is determined for fixed wavelength (in this case, 550 nm), the reflectance can be calculated as a function of the wavelength of an incident light in vacuum. Along the maximum reflectance line, $R=1$ for fixed wavelength of 550 nm as shown in Figure 2, the reflectance of the AFLC cell can be obtained as function of the wavelength of an incident light for given retardation $\gamma_{LC}$ or molecular rotation angle $\theta_{LC}$, i.e., the average optic axis angle of the AFLC molecules in this configuration. In our calculations, the lower path for the line of $R=1$ in Figure 2 is selected since the upper path gives a much narrower range of wavelengths for obtaining high reflectance.

There are two switching directions of the average optic axis in the AFLC layer as shown in Figure 1. Because of the relative orientation of the optic axis of the AFLC layer to that of the WQ wave plate, the optical dispersions of the reflectance in both directions are different from each other. In bidirectional switching modes such as the reflective AFLC mode, the reflectance should be averaged over two switching angles.

Figure 3 shows the optical dispersion of the reflectance averaged over two switching angles in the reflective AFLC cell. For a large tilt angle, the reflectance experiences large optical dispersion. However, the reflectance around $\theta_{LC}=22.5^\circ$ is nearly achromatic for visible light as shown in Figure 3.

We calculate the iso-contrast map for the reflective AFLC cell in the configuration shown in Figure 1. In order to calculate the reflectance for obliquely incident light, the extended $2 \times 2$ Jones matrix formalism [8] was used. The numerical simulations were executed for the switching angle of $24.9^\circ$ ($\gamma_{LC}^{\text{MAX}} \approx 135.6^\circ$), which is identical to the molecular tilt angle of the AFLC material used (CS4001; Chisso Pertochemical Co., Japan). In the bright state, the reflectance was given by the average value over
two switching angles, $\theta_{LC}^{\text{MAX}} = 24.9^\circ$ and $-24.9^\circ$. The reflectance in the dark state was calculated at $\theta_{LC}^{\text{MAX}} = 0^\circ$. The reflectances for the bright and dark states are averaged over wavelengths of red, green, and blue lights to examine the chromaticity.

In Figure 4, the iso-contrast map for a reflective AFLC cell at $|\theta_{LC}| = 24.9^\circ$ is represented in 2-D gray scales. The horizontal axis in Figure 4 is the direction of the polarizer in Figure 1. As shown in Figure 4, the viewing properties are wide and nearly achromatic. The white solid curves represent the inversion lines of contrast at angles between $\pm 55^\circ$ and $\pm 65^\circ$ when viewed from two diagonal directions.

**4. EXPERIMENTAL RESULTS AND DISCUSSION**

The AFLC cell was made using glass substrates coated with indium-tin-oxide. The alignment layer of AL1051 (Japan Synthetic Rubber Co., Japan) was coated on the inner surfaces of the cell and only one of the substrates was rubbed unidirectionally to promote uniform planar alignment. The AFLC material used in this work was CS4001 having the molecular tilt angle of $24.9^\circ$ of Chisso Pertochemical Co. In order to obtain the optimal condition as shown in Figure 3, the cell thickness was maintained using glass spacers of 2 $\mu$m thick. The polarizer and the metal reflector were attached to the AFLC cell as shown in Figure 1.
Figure 5 shows the experimental results and the numerical simulations of the EO reflectance of the AFLC cell as a function of the applied field. Open circles and solid line represent the experimental results and the numerical simulations, respectively. For the simulations, the bilayer model [9], based on the electrostatic dipolar interaction between two nearest neighboring pair dipoles, was employed.

In Figure 5, the reflectance increases slightly below a threshold, which is known as the pretransitional effect [10]. The reflectance increases steeply at 9 V/µm and the bright state is achieved at above 10 V/µm. The field dependence of the measured reflectance is consistent with the simulation results as shown in Figure 5. Using a unipolar square waveform of 20 Hz, the rising and falling times were measured to be about 0.34 and 4.72 ms, respectively. The switching times are fast enough for video-rate applications.

Figure 6 shows the iso-contrast contours (not calibrated) in the reflective AFLC cell with no compensation film under a white illuminating source. In Figure 6, the horizontal axis is the direction of the polarizer. Since the molecular tilt (<45°) appears along the vertical axis, the viewing properties along the y-axis are somewhat narrower than those along the x-axis.


FIGURE 5 Reflectance of the AFLC cell as a function of the applied electric field. Open circles and solid line represent the experimental results and the numerical simulations, respectively.

FIGURE 6 Iso-contrast contours measured in the reflective AFLC cell (not calibrated).
It was found that the experimentally measured viewing properties were consistent with the simulation results as shown in Figure 4.

5. CONCLUSION

We demonstrated the achromatic and wide viewing properties of the reflective AFLC cell in the in-plane optical geometry. Using the $2 \times 2$ Jones matrix formalism, the molecular rotation angle in the AFLC layer and the effective phase retardation through the AFLC layer were optimized in the reflective configuration. Using the WQ wave plate consisting of the HW and QW plates for fixed wavelength of 550 nm, an analytical expression for the retardation of the AFLC layer was obtained as a function of the average optic axis angle of the AFLC molecules. The computational formalism presented here is applicable for various LCD configurations in the in-plane optical geometry. Under the optimal condition, the experimental results for a reflective AFLC cell consisting of a single polarizer and a WQ wave plate were found to agree well with numerical simulations.

REFERENCES